2	3	4	5	6
3	4	5	6	8
4	5	6	8	5
5	7	8	9	3
9	10	9	4	3

Figure 1: Image patch

(1) (10 pts) (a) For the image patch in Figure 1 at the pixel at the center (that is the pixel marked by the black square) apply the following filters and round to the nearest integer value:

i. a 3×3 Gaussian filter $\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

ii. a 3×3 Box filter (that is averaging in a 3×3 neighborhood).

(b) Compute the edge direction and strength (that is the direction and absolute value of the image gradient) at the center pixel using the masks of the Sobel edge detector.

$$S_1 = \frac{1}{8}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad S_2 = \frac{1}{8}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

(c) Apply a median filter to the center pixel. Explain for what kind of noise median filtering will work best.

(d) Why is the Gaussian filter a good smoothing filter? (How can it be implemented fast? How can we implement repeated Gaussian filtering in one operation?)

(e) What happens to the two edges at the boundaries of a dark line on a white background if the image is smoothed with a Gaussian with kernel size larger than the width of the line?

(f) Explain why Box filtering (that is averaging) attenuates the noise.

(g) Explain the concept of aliasing and give an example.

(2) (10 pts) Consider the cube with points $P1, P2, P3, P4, P5, P6, P7, P8$ and 3D coordinates in the world coordinate system as given in Figure 2. A

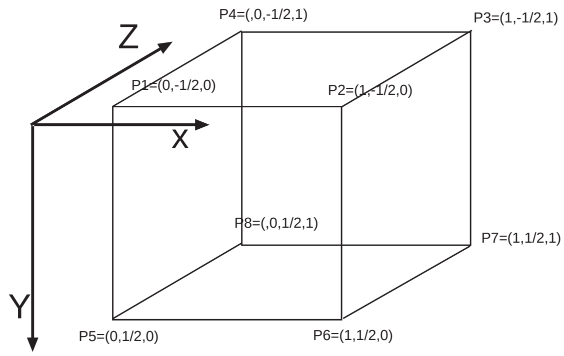

Figure 2:

calibrated camera with focal length $f = 1$ whose origin is at $(0, 0, -3)$ and which has a rotation of $-45°$ around the Y-axis with respect to the world coordinate system takes an image of the cube. The image ccoordinates of the corners of the cube are labeled $p1, p2, p3, p4, p5, p6, p7, p8$ Remember a rotation of angle α around the Y-axis can be expressed by the rotation matrix $R = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$, and $\cos(-45°) = \frac{1}{2}\sqrt{2}$, and $\sin(-45°) = -\frac{1}{2}\sqrt{2}$.

(a) Derive the projection matrix mapping homogeneous world coordinates to homogeneous image coordinates.

(b) Compute the homogeneous and the non-homogenous image coordinates of points $p5, p6, p7, p8$.

(c) Derive the non-homogenous coordinates of the 3 vanishing points, corresponding to the 3 parallel lines.

(d) Compute the vanishing point of the line $P5\bar{P}7$

(e) How would the camera need to be positioned with respect to the cube, such that 2 of the vanishing points are ideal (that is are at infinity)?

(3) (5pts) Describe the Canny edge detector. Exlain its three modules.

(4) (5 pts) Consider a bright patch of $0.5m^2$ at the floor. The patch is due to a point light source of 100W/sr radient intensity, which is 10m above the patch.

(a) Compute the solid angle from the light source to the patch.

(b) What is the irradiance of the patch on the floor?

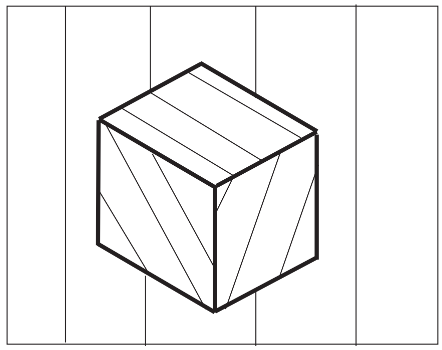

Figure 3:

2	3	4	5	6
3	4	5	6	8
4	5	6	8	5
5	7	8	9	3
9	10	9	4	3

at t_0

2	2	3	5	6
2	3	4	5	7
4	4	5	6	8
4	5	7	9	9
9	9	10	10	5

at t_1

Figure 4: Image patches at two instances in time

(5) (5pts) (a) Consider the cube with a stripe pattern in front of a plane with a stripe pattern shown in Figure 3. The scene is observed by a moving camera which translates to the right (parallel to the x-axis). Draw the optical flow field and draw the normal flow field.

(b) At time t_0 and time t_1 the camera observes the image patches shown in Figure 4 (the image patch at t_0 is the same as the one in problem 1). Compute the normal flow vector at the center of the patch. Estimate the spatial derivative I_x and I_y by averaging the derivatives computed with the Sobel operator at the two time instances. Compute the time derivative I_t as difference between the intensity at the center pixels.