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NAVIGATION WITH UNCERTAINTY: II. Avoiding High Collision Risk Regions

Philippe Burlina
Daniel DeMenthon
Larry S. Davis

Computer Vision Laboratory
Center for Automation Research
University of Maryland
College Park, MD 20742-3411

Abstract

We present a probabilistic method for sensor based robotic navigation in dynamic and noisy environments. The method generates a trajectory that guarantees a tolerable associated collision risk. Estimates of the obstacle's kinematic parameters and measures of confidence in these estimates are used to produce regions where the probability of encountering any obstacle is bounded by a predefined value.

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1 Introduction

1.1 Methods in Probabilistic Navigation

We are studying probabilistic approaches to sensor-based navigation in noisy dynamic environments. Here, we address the general problem of trajectory planning where an autonomous robot must reach a moving goal point and avoid surrounding moving obstacles. The motions of the obstacles are not known *a priori*, but are recovered through noisy sensor *observations*.

Due to the stochastic nature of the problem, it is in most cases irrelevant to search for a trajectory guaranteed to be free of collisions. Our approach is, instead, to derive a trajectory to the goal that guarantees a certain level of collision safety for the autonomous robot. Here we describe a stochastic framework in which measures of uncertainty on the obstacle's motion are used to derive safe trajectories toward the goal. Our approach is independent of the kind of sensor or object motion recovery scheme used by the robotic system.

The path planning problem in known static environments has been extensively addressed in the past. Methods for addressing this problem have primarily relied on purely geometric tools using the idea of configuration space and Voronoi diagrams [16, 17] and on field-optimization methods [11]. Variants on these methods have been proposed to include certain cases of path planning in known dynamic environments [7]. Other approaches made use of probabilistic techniques for learning planned paths by updating transition probabilities in a state graph [6].

Much less has been done in the case of unknown static environments. In this case an important tool is the *occupancy grid*, used for sensory data fusion and navigation [5, 12, 13]. The system keeps a map of the space and each space-cell's probability of occupancy is calculated based on models of the sensors' uncertainties. Bayes' rule is used for updating this probability when new sensor readings are collected for a particular neighborhood in the space. Another interesting and related approach is given in [2]: a polar histogram representing a *polar obstacle density* in each direction is compiled from range readings. A similar approach to obstacle avoidance for an autonomous helicopter is proposed in [4]. Still less has been done in the case of unknown dynamic environments. *Collision zones* are used in [10]: simple cases are investigated where moving obstacles are replaced by stationary collision zones as input to classic collision avoidance algorithms. Stochastic automata are proposed for manipulators in noisy environments [14]. Other approaches to this problem include [1, 9].

We propose two different and complementary approaches to probabilistic navigation. The first method, described in [3], addressed the problem of reaching a goal point in a region of high collision risk. This method involves deriving for every destination point of interest at a given time an associated probability of collision and a probability of being operational. Trajectory planning is then reduced to the dynamic optimization of a cost function including these measures.

A second approach, introduced in this paper, addresses the problem of avoiding high collision risk regions entirely, we are no longer interested in the safety of a given destination point in space but rather in the overall safety of an entire region of destinations.

The idea is to identify a region of space in which the probability of encountering any obstacle is kept at each instant less than a predefined tolerable value. For a given threshold value many such regions may be identified. For the sake of presentation, two examples of the construction of the *clear region* are considered here: assuming the region to be the complement of a disk (surrounding the moving obstacle), or the complement of an ellipse grown by a circle.

The paper is organized as follows. Section 2 defines the problem and the objectives, gives the assumptions and the basic probabilistic model. The construction of a *clear region* is presented for a unique obstacle in Section 3. A valid *clear region* is constructed as the complement of a circle in Section 3.2. Construction outside an augmented ellipse is shown in Section 3.3. A method for finding the largest circular clear region is presented in Section 3.4. Section 4 extends the construction to the case of multiple obstacles. The navigation strategy making use of the *clear subsets* is presented in Section 5.

2 Problem Definition, Models and Assumptions

In the following we summarize the basic assumptions of the problem. We assume the presence of more than one obstacle. For any obstacle, the only information known a priori is:

- its structure, i.e. the radius r_e of the smallest circle enclosing the obstacle.
- bounds on its speed and normal/tangential acceleration.

The precise motion of the object is unknown. It may combine rotations and translations. The proposed approach makes the following geometrical and differential simplifications:

- The world is two dimensional.
- The trajectory of the robot is approximated as a polyline in the plane; between any two successive vertices of this polyline, a constant time interval δt has elapsed, during which the robot's displacement is at a constant velocity. At any vertex the robot can change its velocity (direction and speed) within limits defined by mechanical and inertial constraints.
- The obstacle object may be represented by a not necessarily convex polygon. The robot is a point.

Based on this framework, the problem consists of determining a time-space region of safe destinations.

A Kalman filter is used to provide the necessary mean and covariance of the obstacle's motion state probability distribution. For the purpose of calculating our safe regions we are interested only in the translation components of the obstacle's motion. The components of the kinematic state vector are the position coordinates $[x_n, y_n]^T$ in the global reference system and their first and second derivatives, so as to model the object's trajectory by trajectory elements with piecewise constant accelerations. The kinematic state evolution of the object is modeled locally as a first order autoregressive time invariant model

$$S_{n+1} = AS_n + V_n \tag{1}$$

where

- $S_n = [U_n]^T$, the object's motion state vector,
- $U_n = \left[X_n, \dot{X}_n, \ddot{X}_n\right]^T$, where $X_n, \dot{X}_n, \ddot{X}_n$ are respectively the position, velocity and acceleration vectors of the center of the obstacle's enclosing circle of radius r_e,
- $X_n = [x_n, y_n]^T$, the position of the center of the enclosing circle,
- A = the model transition matrix.

In the remainder of this report we will refer to the center of the disk enclosing the obstacle simply as the *obstacle's center*. The model approximates the motion of the center as a uniformly accelerated movement,

$$A = \begin{bmatrix} E_1 + E_2 + \frac{1}{2}E_3 \\ E_2 + E_3 \\ E_3 \end{bmatrix} \tag{2}$$

where we define E_1, E_2, E_3 to be respectively

$$E_1 = \begin{bmatrix} I_{2\times 2} & 0_{2\times 2} & 0_{2\times 2} \end{bmatrix}, \tag{3}$$

$$E_2 = \begin{bmatrix} 0_{2\times 2} & I_{2\times 2} & 0_{2\times 2} \end{bmatrix}, \tag{4}$$

$$E_3 = \begin{bmatrix} 0_{2\times 2} & 0_{2\times 2} & I_{2\times 2} \end{bmatrix}. \tag{5}$$

The motion state S_n is not accessible directly, but is instead derived from observations obtained from sensor readings. The equation relating the state vector S_n and the observation vector O_n is given by

$$O_n \quad = \quad B(S_n) + W_n \tag{6}$$

where

- O_n = the observation vector at time step t_n,
- $O^n = (O_1, O_2, \ldots, O_n)$, all the observations made up to time t_n,
- $W_n =$ a zero mean Gaussian distributed independent error sequence on the observations with covariance matrix equal to Ω_k, and independent of the model error V_n,
- $B(.)$ = the observation function.

Based on the past and present observations O^n, the Kalman filtering method provides an optimal estimate of the present time object motion state $S_{n|n}$ and an optimal prediction of the object motion state k steps in the future $S_{n+k|n}$, along with their respective covariance matrices $\Sigma_{n|n}$ and $\Sigma_{n+k|n}$. Let us now turn to the derivation of the safe region.

3 The Clear Region

As introduced previously, the idea is to construct a region in space where the risk is less than a predefined acceptable value. Suppose that we are at present time t_n. At every time instant $t \geq t_n$, we construct a region $\Re(t)$ such that the probability that any part of any obstacle may be anywhere inside this region at time t is less than a predefined threshold PT. We call this region the *PT-clear region*. We define $\Re_{n+k,n+k+1} \in R^3 \times [t_{n+k}, t_{n+k+1}]$ as the PT-clear region for t ranging over the time interval $[t_{n+k}, t_{n+k+1}]$. By navigating inside the PT-clear subset we guarantee a certain level of collision safety for the robot: if no part of any obstacle may be encountered anywhere in this region at time t, there cannot be any collision at this time if the robot remains entirely inside this region.

Since we only have a discrete model of the motion evolution of the obstacles, and thus only a discrete-time stochastic description of the obstacle positions, we can really only build an approximation of $\Re_{n+k,n+k+1}$.[1] The subset $\Re_{n+k,n+k+1}$ is thus approximated as the interpolation over the time interval $[t_{n+k}, t_{n+k+1}]$ of $\Re(t_{n+k})$ and $\Re(t_{n+k+1})$. We now turn to the construction of $\Re(t_{n+k})$ in the case of a single obstacle i, which we denote by $\Re^i_{n+k}$.

We set an upper bound PT_i on the probability that any part of obstacle i may be anywhere inside $\Re^i_{n+k}$ at time t_{n+k}, and we construct $\Re^i_n$ so as to satisfy this bound. One way of constructing it is by constructing a disk around the estimated position of the obstacle's center $X_{n+k|n}$ such that the probability that any part of the obstacle is outside this disk is less than PT_i. Then $\Re^i_{n+k}$ is taken to be the complement of this disk.

3.1 Circular Construction

Consider the present instant t_n. We derive here a PT_i-clear region k time steps into the future $\Re^i_{n+k}$ in the case of a single obstacle i using a circular construction. The center of the obstacle k steps into the future, X_{n+k}, is a random variable and we have shown how to derive the mean and covariance of its distribution. We want to find $B(X_{n+k|n}, r_c)$, the disk of radius r_c centered on the predicted position of the obstacle center $X_{n+k|n}$, such that the probability of any point of the obstacle being outside this disk is less than a predefined probability threshold PT_i (Figure 1). The following result gives a radius that satisfies the bound.

We have

$$P\{\text{any point of obstacle } i \text{ outside } B(X_{n+k|n}, r_c)\} \leq PT_i. \tag{7}$$

If

$$r_c = \sqrt{\frac{tr(\Sigma_{X_{n+k|n}})}{PT_i}} + r_e \tag{8}$$

where $tr(.)$ denotes the trace, $\Sigma_{X_{n+k|n}}$ is the covariance matrix of the position components of the obstacle motion state, and r_e is the radius of the smallest disk

[1] Having a continuous-time model approximation of the obstacle motion would enable us to construct $\Re(t)$ at the cost of handling a diffusion equation for the motion.

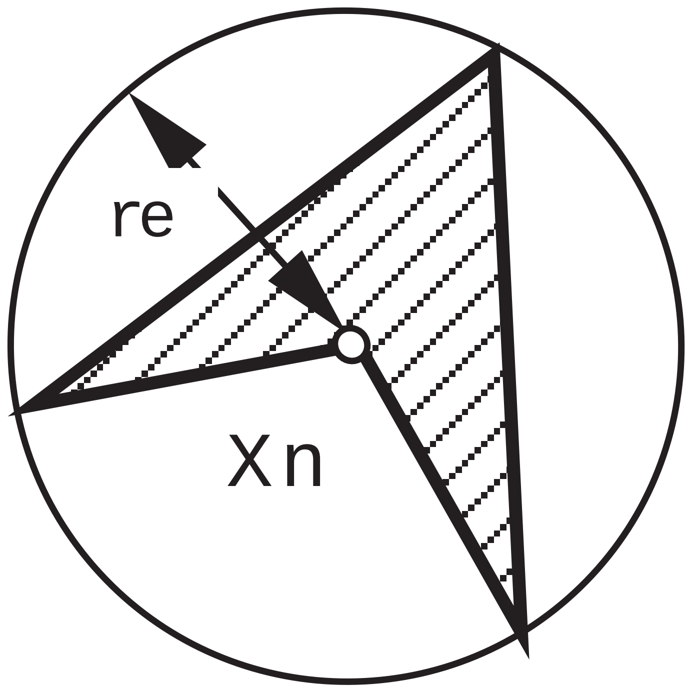

Figure 1: Obstacle outside the clear radius disk

centered at the center of the obstacle and completely enclosing the obstacle (see Figure 1).

Proof: As a necessary condition for any point of the obstacle to be outside the disk $B(X_{n+k|n}, r_c)$, X_{n+k} must be outside the disk $B(X_{n+k|n}, r_c - r_e)$ (Figure 1). The converse, however, is not necessarily true. Thus we have

$$\begin{aligned} &P\{\text{any point of the obstacle outside } B(X_{n+k|n}, r_c)\} \\ &\leq P\{X_{n+k} \notin B(X_{n+k|n}, r_c - r_e)\} \end{aligned} \tag{9}$$

since the event of the LHS is included in the event of the RHS. The bound gets tighter with the *roundness* of the obstacle. We also have

$$\begin{aligned} &P\{X_{n+k} \notin B(X_{n+k|n}, r_c - r_e)\} \\ &= P\{(X_{n+k} - X_{n+k|n})^T (X_{n+k} - X_{n+k|n}) \geq (r_c - r_e)^2\} \\ &= P\{[X^2 + Y^2] \geq (r_c - r_e)^2\} \end{aligned} \tag{10}$$

where we define X and Y to be respectively

$$\begin{aligned} X &= (x_{k+n} - x_{k+n|k}) \\ Y &= (y_{k+n} - y_{k+n|k}) \end{aligned}$$

It is now sufficient to find a bounded value of r_c for which we have

$$P\{X_{n+k} \notin B(X_{n+k|n}, r_c - r_e)\} \leq PT_i. \tag{11}$$

so as to satisfy equation (7). For any positive random variable U we have (Markov Bound):

$$P\{U \geq a\} \leq \frac{E\{U\}}{a}. \tag{12}$$

Then using (12) we have

$$\begin{aligned} P\{\left[X^2 + Y^2\right] \geq (r_c - r_e)^2\} &\leq \frac{E[X^2 + Y^2]}{(r_c - r_e)^2} \\ &\leq \frac{\sigma^2_{x_{n+k|n}} + \sigma^2_{y_{n+k|n}}}{(r_c - r_e)^2} \end{aligned}$$

The previous inequality is satisfied and thus equation (11) is satisfied if we let

$$PT_i = \frac{\sigma^2_{x_{n+k|n}} + \sigma^2_{y_{n+k|n}}}{(r_c - r_e)^2} \tag{13}$$

This yields

$$\begin{aligned} r_c &= \sqrt{\frac{\sigma^2_{x_{n+k|n}} + \sigma^2_{y_{n+k|n}}}{PT_i}} + r_e \\ &= \sqrt{\frac{tr(\Sigma_{X_{n+k|n}})}{PT_i}} + r_e \end{aligned} \tag{14}$$

where $tr(A)$ denotes the trace of the matrix A.

This result is intuitive: the radius r_c is an increasing function of the trace of the obstacle center position covariance matrix and a decreasing function of the probability threshold PT_i. Next we construct the clear region as a complement of an augmented ellipse.

3.2 Elliptic Construction

In the case of a significant difference between the uncertainties along each of the axes, we find a less conservative clear region by taking the complement of an augmented ellipse. We have the following result:

> Let E_g be the ellipse centered on the predicted position of the obstacle's center $X_{n+k|n}$ with axes in the directions of the eigenvectors of the covariance matrix $\Sigma_{X_{n+k|n}}$ and equal to $\frac{\sigma_{x_{n+k}}\sqrt{2}}{\sqrt{PT_i}}, \frac{\sigma_{y_{n+k}}\sqrt{2}}{\sqrt{PT_i}}$, grown by the disk of radius r_e (see Figure 3); then
>
> $$P\{\text{any point of obstacle } i \text{ outside } E_g \, at \, t_{n+k}\} \leq PT_i. \tag{15}$$

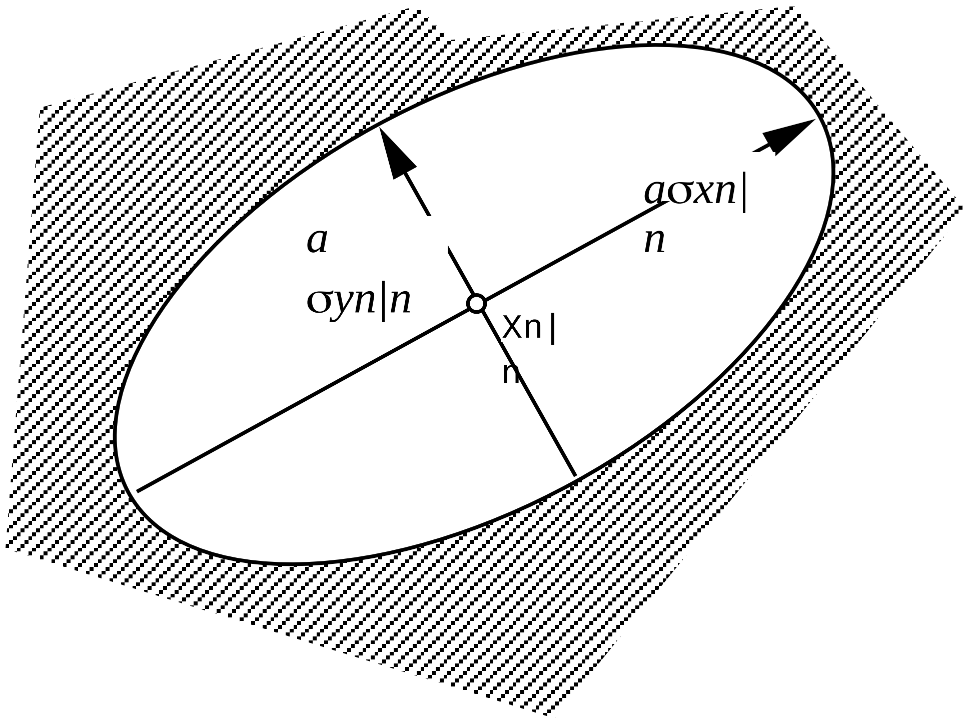

Figure 2: Ellipse E_a.

Proof: Consider the ellipse defined by

$$E_a = \{X_{n+k} | \left(X_{n+k} - X_{n+k|n}\right)^t \Sigma^{-1}_{X_{n+k|n}} \left(X_{n+k} - X_{n+k|n}\right) = a^2\}. \tag{16}$$

with principal axes in the direction of the eigenvectors of the matrix $\Sigma^{-1}_{X_{n+k|n}}$ and therefore also of the matrix $\Sigma_{X_{n+k|n}}$ (the basis vectors), and major and minor axes equal to the standard deviations on the axes (the square roots of the eigenvalues) multiplied by the factor a. This ellipse is centered on the estimated obstacle center $X_{n+k|n}$ (Figure 2). We now want to find a as a function of PT_i so as to construct a PT_i-clear region as $(E_a)^c$, the complement of E_a. Suppose that we give an upper bound PT_i on the probability of the obstacle center X_{n+k} being outside the ellipse E_a

$$P\{X_{n+k} \notin E_a\} \leq PT_i. \tag{17}$$

Using the Markov bound again we have

$$\begin{aligned} P\{X_{n+k} \notin E_a\} &= P\{(X_{n+k} - X_{n+k|n})^t \Sigma^{-1}_{X_{n+k|n}} (X_{n+k} - X_{n+k|n}) \geq a^2\} \\ &\leq \frac{E\{(X_{n+k} - X_{n+k|n})^t \Sigma^{-1}_{X_{n+k|n}} (X_{n+k} - X_{n+k|n})\}}{a^2} \\ &\leq \frac{2}{a^2} \end{aligned}$$

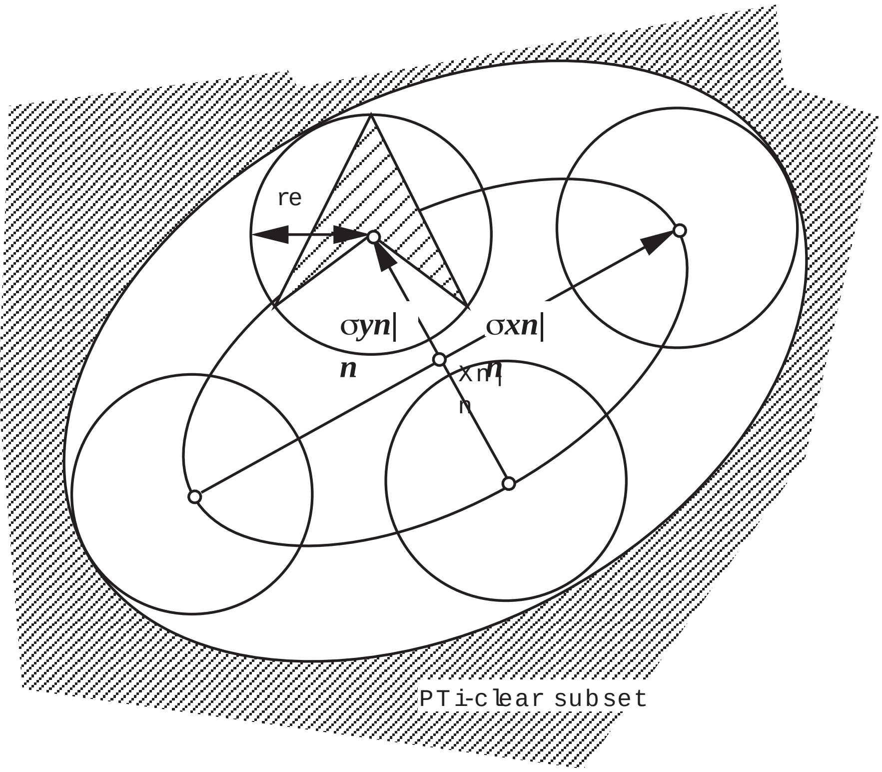

Figure 3: Clear Region constructed as the complement of an augmented ellipse

Thus to satisfy the bound in equation (17) we can take

$$a = \frac{\sqrt{2}}{\sqrt{PT_i}}.$$

Now consider the region equal to the original ellipse $E_{\sqrt{\frac{2}{PT_i}}}$ "grown" by the disk of radius r_e. This region is in fact E_g. If any point of the obstacle i is outside E_g, then necessarily the obstacle's center X_n must be outside $E_{\sqrt{\frac{2}{PT_i}}}$. The probability that the center of the obstacle is anywhere outside $E_{\sqrt{\frac{2}{PT_i}}}$ is greater than the probability that any part of the obstacle is outside of a region E_g. A valid clear region may thus be obtained by taking the complement of the ellipse centered on the predicted position of the obstacle's center and with major and minor axes equal to $\frac{\sigma_{x_{n+k}}}{\sqrt{PT_i}}\sqrt{2}, \frac{\sigma_{y_{n+k}}}{\sqrt{PT_i}}\sqrt{2}$ grown by the disk of radius r_e (see Figure 3).

Note that these two results are obtained without any assumption about the motion distribution; to find a valid clear region we have used the Markov bound. Therefore only the mean and variance of the position components were used and not their distribution. Note also that the Markov bound is not very tight; this may lead to a conservative clear region. Consequently, we would like to get the largest clear region for a given probability threshold. We show how to find it in the case of the circular construction.

3.3 Largest Clear Region

Let us assume at this point that the conditional probability distribution of the motion state vector conditioned by the observations $f_{S_n|O^n}(.)$ is Gaussian, with mean $S_{n|n}$ and covariance $\Sigma_{n|n}$, i.e.

$$f_{S_n|O^n}(S) = N_{(S_{n|n},\Sigma_{n|n})}(S)$$

where $N(E[Y],\Sigma^Y)(.)$ denotes the Gaussian distribution having mean $E[Y]$ and covariance Σ^Y, i.e.

$$N_{(E[Y],\Sigma^Y)}(Y) = \frac{1}{\sqrt{(2\pi)^p \det(\Sigma^Y)}} \exp \frac{-(Y-E[Y])^T(\Sigma^Y)^{-1}(Y-E[Y])}{2}.$$

where p is the dimension of Y. Similarly, the object's kinematic parameters S_{n+k} at time t_{n+k} conditioned on the observations made up to the present time are Gaussian with mean equal to the predicted state $S_{n+k|n}$, and with covariance matrix equal to the error covariance $\Sigma_{n+k|n}$, i.e.

$$f_{S_{n+k}|O^n}(S) = N_{(S_{n+k|n},\Sigma_{n+k|n})}(S).$$

In the case of a linear observation function $B(.)$, this assumption is exact and is a consequence of having assumed Gaussian errors in the motion and observation equations. This Gaussian assumption corresponds only to a working approximation in the case of a nonlinear observation function $B(.)$, but it is a satisfactory approximation since the unconditioned distribution of the motion state vector is Gaussian as a consequence of having assumed an autoregressive motion model corrupted by Gaussian noise.

In the case of the circular construction the largest clear region is found for the value of r_c for which we have equality in (11) or equivalently

$$P\{(X_{n+k}-X_{n+k|n})^t(X_{n+k}-X_{n+k|n}) \geq (r_c-r_e)^2\} = PT_i. \tag{18}$$

We give two methods for calculating the value of the radius r_c for which we obtain equality in the equation 11. Consider the function

$$P(r) = P\{(X_{n+k}-X_{n+k|k})^t(X_{n+k}-X_{n+k|k}) \leq (r-r_e)^2\}. \tag{19}$$

Our problem is to find the value r_c^* for which $P(r_c^*) = 1 - PT_i$. Clearly there is a unique solution to this problem since $P(.)$ is a strictly monotone increasing function. Therefore we may use the Newton method to find a solution. We have

$$\begin{aligned} r_c^{n+1} &= r_c^n - \frac{P(r_c^n)-(1-PT_i)}{\frac{d(P(r_c^n))}{dr_c}} \\ &= r_c^n + \frac{-P(r_c^n)+1-PT_i}{2r_c^n f_Y((r_c^n-r_e)^2)} \end{aligned} \tag{20}$$

where f_Y denotes the conditional density function of the squared distance

$$Y = (X_{n+k}-X_{n+k|k})^T(X_{n+k}-X_{n+k|k})$$

The initial value r_c^0 can be taken to be any value in the interval $[r_e, r_e + \sqrt{\frac{tr(\Sigma_{X_{n+k|n}})}{PT_i}}]$ since we have

$$P(r_e) = 0 \tag{21}$$

and from the previous subsection we know that

$$P(r_e + \sqrt{\frac{tr(\Sigma_{X_{n+k|n}})}{PT_i}}) \geq 1 - PTi \tag{22}$$

Let us now turn to the calculation of $f_Y(.)$. Consider the distribution of the square of a Gaussian random variable $A = Z^2$ (see [8, 15]). This distribution is chi-square and given by

$$f_A(a) = \frac{1}{\sqrt{a}} \frac{\exp^{-\frac{a}{2\sigma_Z^2}}}{\sqrt{2\pi\sigma_Z^2}}. \tag{23}$$

We are interested in the distribution of Y, the sum of squares of two random variables. This distribution is chi-square with two degrees of freedom. We obtain an explicit formula by recalling that the distribution of the sum of two independent random variables (we assume here that we work in the eigenbasis of the covariance matrix) is the convolution of the distributions of these random variables. Using this leads to

$$f_Y(t) = \frac{\exp^{\frac{-t}{2\sigma_y^2}}}{2\pi\sigma_x\sigma_y} \int_0^t \frac{\exp^{\frac{-x|\sigma_y^2 - \sigma_x^2|}{2\sigma_x^2\sigma_y^2}}}{\sqrt{t-x}\sqrt{x}} dx$$

where

$$\sigma_x^2 = \sigma_{x_{n+k|n}}^2$$
$$\sigma_y^2 = \sigma_{y_{n+k|n}}^2.$$

As an alternative method, noting that the solution lies in the interval $[r_e, r_e + \sqrt{\frac{tr(\Sigma_{X_{n+k|n}})}{PT_i}}]$ (equations (21) and (22)), an alternative method would be to use a simple dichotomous search in this interval, thus avoiding the calculation of the density. A good approximation to the function $P()$ can be determined by one dimensional integration of the Gaussian error function, i.e.[2]

$$P(r) = \int_{-(r-r_e)}^{(r-re)} N_{(x_{n+k|n}, \sigma_{x_{n+k|n}})}(Q(\frac{\sqrt{(r-r_e)^2 - x^2}}{\sigma_{y_{n+k|n}}}) - Q(-\frac{\sqrt{(r-r_e)^2 - x^2}}{\sigma_{y_{n+k|n}}}))dx \tag{24}$$

where Q is the readily tabulated complement error function

$$Q(x) = \int_{-\infty}^{x} N_{(0,1)}(t)dt.$$

[2]Gaussian density integrated on the disk. Again we assume that the covariance matrix is diagonal.

4 The Clear Region; the Case of Multiple Obstacles

The extension to the case where we have many obstacles follows naturally. Consider $R(t)$ the subspace of points reachable at time t by the robot given the dynamical constraints. An obstacle is considered reachable at time t if any part of this obstacle is predicted to be in the reachable region $R(t)$. We redefine the PT-clear region $\Re(t)$ to be the subset of points such that the probability that any reachable obstacle at time t is inside the region $\Re(t)$ is less than PT. This redefinition reflects the idea that obstacles that are not reachable have no influence on the collision risk, and therefore no influence on the clear region. The following result shows how to construct a clear region with respect to many obstacles.

> Let there be q reachable obstacles at time t_{n+k}, with independent motions. For each obstacle i, $i = 1..q$, let $\Re^i_{n+k}$ be the PT_i - clear region with respect to the ith obstacle. If
> $$PT_i = 1 - (1 - PT)^{\frac{1}{q}} \tag{25}$$
> and
> $$\Re_{n+k} = \bigcap_{i=1,..q} \Re^i_{n+k} \tag{26}$$
> then
> $$P\{\text{any point of any of the } q \text{ obstacles } \in \Re_{n+k} at\, t_{n+k}\} \leq PT. \tag{27}$$

Proof: We now show that the choice of PT_i in (25) is indeed suitable to satisfy (27). Equation (27) is equivalent to

$$P\{\text{no point of any obstacle } \in \Re_{n+k}\} \geq (1 - PT). \tag{28}$$

Calling $(\Re^i_p)^c$the complement set of $\Re^i_p$ we get

$$P\{\bigwedge_{all\, i=1..p} (\text{all points of obstacle } i \in (\Re^i_p)^c)\} \leq P\{\text{no point of any obstacle } \in \Re_{n+k}\}. \tag{29}$$

By construction, each PT_i clear region $\Re^j_{n+k}, j = 1..q$ satisfies

$$P\{\text{any point of obstacle } i \in \Re^i_{n+k}\} \leq PT_i, \tag{30}$$

which is equivalent to

$$P\{\text{all points of obstacle } i \in \Re^i_{n+k}\} \geq (1 - PT_i). \tag{31}$$

Using the assumption that the obstacle's movements are independent, and taking

$$PT_i = 1 - (1 - PT)^{\frac{1}{q}} \tag{32}$$

we have

$$P\{\text{no point of any obstacle } \in \Re_{n+k}\} \;\; \geq \;\; P\{\bigwedge_{all\, i=1..p} (\text{all of obstacle } i \in (\Re^i_p)^c)\}$$

$$
\begin{aligned}
&= \prod_{i=1,..q} P\{\text{all of obstacle } i \in \Re^i_{n+k}\} \\
&\geq \prod_{i=1,..q} (1 - PT_i) \\
&= \prod_{i=1,..q} (1 - PT)^{\frac{1}{q}} \\
&= 1 - PT.
\end{aligned}
$$

showing that the choice of the intersection over all the PT_i clear regions along with PT_i as in (25) does indeed satisfy the bound (28).

5 Navigation Strategy

At the present time instant, the predictor gives the best prediction k steps into the future of the positions of the obstacles that have been monitored and tracked, for $k = 1, 2, ..N$. The PT-clear regions $\Re_{n+k}$ are constructed for each prediction instant. The PT-clear region $\Re(t)$ for $t \in [t_1, t_N]$ is obtained by interpolation. Consider the instantaneous reachable subset $R(t)$. We define the "navigable" subset $N(t)$ to be the intersection of $R(t)$ and $\Re(t)$. The horizon N for the prediction can be chosen to be the time instant at which the error in the prediction is large enough so that the navigable subset is reduced to the void subset. A navigation strategy would be to follow rectilinear trajectories to the goal point when this is possible, (this would indeed correspond to uniform rectilinear trajectories) combined with trajectories following the border of $N(t)$ when the corresponding rectilinear trajectory would leave $N(t)$ (Figure 4). Since all destinations in $N(t)$ are reachable we construct a trajectory to the goal point that is both acceptable in terms of dynamical constraints and ensures an acceptable level of collision risk.

We leave aside the problem of determining the PT_i, and suggest only that their choice can be learned through experience if this is allowed in the given navigation context; otherwise, it can be determined through simulation techniques used along with a suitable learning scheme.

6 Conclusion

There are many situations in robotic satellite maintenance, autonomous vehicle guidance, and autonomous industrial robot guidance, in which risk assessment is essential.

The paper has presented an approach to probabilistic sensor-based navigation in noisy and dynamic environments. In this approach, measures of confidence on the motion estimates are used to produce regions with tolerable associated risk levels. We have shown how to produce particular instances of such regions in the cases of a single obstacle and multiple obstacles.

In another approach, estimates of the obstacle motion and measures of confidence in these estimates induce a probability of collision associated with each robot displacement used for optimizing the trajectory of the robot. This approach was presented in [3]. These two approaches are complementary and can be used simultaneously. The approaches can be

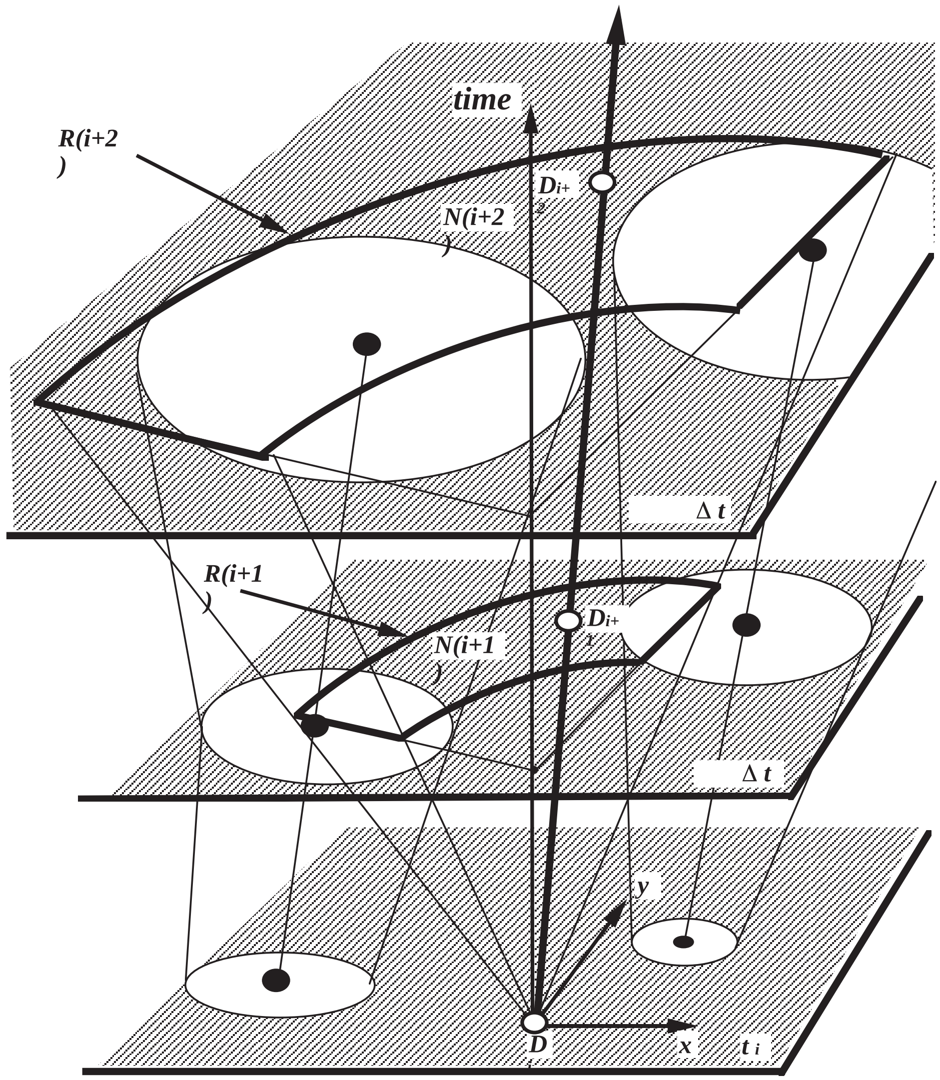

Figure 4: Navigable subset

extended to planning in situations in which a robot works in a hostile environment containing several moving obstacles or possibly hostile agents. As a possible strategy, the robot may approach one of the agents by optimizing its trajectory with respect to the probability of intercepting this agent, while maintaining a tolerable level of safety with respect to this agent, using the first approach ([3]). Meanwhile the robot stays clear of the other agents, using the second approach, as described in this report. Practically, the second approach could provide spatial constraints as input to the optimization performed in the first approach.

The proposed probabilistic framework can be integrated into a low level trajectory controller that could operate with a high level planner. The planner may provide navigational subgoals along with their acceptable degrees of risk.

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