Image formation 2
Blur circle

Points at distance $-z$ are brought into focus at distance $z'$. A point at distance $-\bar{z}$ is imaged at point $\bar{z}'$ from the lens

\[
\frac{1}{\bar{z}'} + \frac{1}{-\bar{z}} = \frac{1}{f}
\]

and so

\[
\bar{z}' - z' = \frac{f}{(\bar{z} + f)} \frac{f}{(z + f)} (\bar{z} - z)
\]

Thus points at distance $-\bar{z}$ will give rise to a blur circle of diameter

\[
b = \frac{d}{\bar{z}'} |\bar{z}' - z'|\]

with $d$ the diameter of the lens
Irradiance, $E$

- Light power per unit area (watts per square meter) incident on a surface.
- If surface tilts away from light, same amount of light strikes bigger surface (less irradiance) (no foreshortening)
- $E$ times pixel area times exposure time -> pixel intensity
Radiance, L

• Amount of light radiated from a surface into a given solid angle per unit area (watts per square meter per steradian).

• Note: the area is the foreshortened area, as seen from the direction that the light is being emitted.

• **Brightness corresponds roughly to radiance**
Solid angle

- The solid angle subtended by a cone of rays is the area of a unit sphere (centered at the cone origin) intersected by the cone.
- A hemisphere covers $2\pi$ sterradians.
What’s the solid angle subtended by this patch, area $A$, seen from $P$?

Multiply by $\cos(\theta)$ to account for foreshortening.

Divide by $R^2$ to convert the area to what you’d see on a unit sphere.

\[ \frac{A \cos(\theta)}{R^2} \]
Pixel Brightness and Scene Brightness

\[
d\omega = \frac{d\alpha \cos \alpha}{(f / \cos \alpha)^2} = \frac{dA \cos \theta}{(Z / \cos \alpha)^2} \Rightarrow dA = \frac{\cos \alpha}{\cos \theta} \left( \frac{Z}{f} \right)^2 d\alpha \\
\Omega = \frac{\pi}{4} \left( \frac{D^2}{f} \right) \cos^3 \alpha \cos \theta \\
P = L dA \Omega \cos \theta \Rightarrow dP = L dA \frac{\pi}{4} \left( \frac{D}{Z} \right)^2 \cos^3 \alpha \cos \theta \\
E = \frac{dP}{da} = L \frac{dA}{da} \frac{\pi}{4} \left( \frac{D}{Z} \right)^2 \cos^3 \alpha \cos \theta \Rightarrow E = \frac{\pi}{4} \left( \frac{D}{f} \right)^2 \cos^4 \alpha \ L \\
\square E = k \ L
\]
Relationship: Image Irradiance and Scene Radiance

\[ E = L \frac{\pi}{4} \left( \frac{D}{f} \right)^2 \cos^4 \alpha \]

\[ E = L \frac{\pi}{4} \left( \frac{D}{f} \right)^2 \cos^4 \alpha \]
Coordinate system

Horn, 1986
Radiosity

The total power leaving a point on a surface per unit area on the surface

\[ B(P) = \int_{\Omega} L(P, \theta, \phi) \cos \theta d\Omega \]

If radiance independent of angle -> integrate over hemisphere

\[ B(P) = L(P) \int_{0}^{2\pi} \int_{0}^{\pi/2} \cos \theta \sin \theta \, d\phi \, d\theta = \pi L(P) \]
BRDF

Horn, 1986

Figure 10-7. The bidirectional reflectance distribution function is the ratio of the radiance of the surface patch as viewed from the direction \((\theta_e, \phi_e)\) to the irradiance resulting from illumination from the direction \((\theta_i, \phi_i)\).

\[
BRDF = f(\theta, \phi, \theta_e, \phi_e) = \frac{L(\theta_e, \phi_e)}{E(\theta, \phi)}
\]

\[
= \frac{L_e(\theta_e, \phi_e)}{L_i(\theta, \phi) \cos \theta d\omega}
\]

unit: \(sr^{-1}\)
Special Cases: Lambertian

$$f(\theta_i, \phi_i, \theta_e, \phi_e) = \rho \frac{1}{\pi}$$

Note: reflected light is with strength proportional to \( \cos \) of angle with surface normal, but the area is foreshortened

- Albedo is fraction of light reflected.
- Diffuse objects (cloth, matte paint).
- Brightness doesn’t depend on viewpoint.
- Does depend on angle between light and surface.

$$L(\theta_e, \phi_e) \propto \cos(\theta)$$
Lambertian Examples

Scene
(Oren and Nayar)

Lambertian sphere as the light moves.
(Steve Seitz)
Another important class of surfaces is specular, or mirror-like.

- radiation arriving along a direction leaves along the specular direction
- reflect about normal
- some fraction is absorbed, some reflected
- on real surfaces, energy usually goes into a lobe of directions

Specular surfaces

- Brightness depends on viewing direction.
Phong’s model

• Vision algorithms rarely depend on the exact shape of the specular lobe.

• Typically:
  – very, very small --- mirror
  – small  --- blurry mirror
  – bigger --- see only light sources as “specularities”
  – very big --- faint specularities

• Phong’s model
  – reflected energy falls off with \( \cos^n(\delta \theta) \)

(Forsyth & Ponce)
Lambertian + Specular Model

\[ L(P, \theta_o, \phi_o) = \rho_d(P) \int_{\Omega} L(P,\theta_i,\phi_i) \cos \theta_i d\Omega \]
\[ + \rho_s(P)L(P, \theta_s, \phi_s) \cos^n(\theta_s - \theta_o) \]
Lambertian + specular

- Two parameters: how shiny, what kind of shiny.
- Advantages
  - easy to manipulate
  - very often quite close true
- Disadvantages
  - some surfaces are not
    - e.g. underside of CD’s, feathers of many birds, blue spots on many marine crustaceans and fish, most rough surfaces, oil films (skin!), wet surfaces
  - Generally, very little advantage in modelling behaviour of light at a surface in more detail -- it is quite difficult to understand behaviour of L+S surfaces (but in graphics???)
Lambertian + Specular + Ambient

Human Eye

- pupil: 1-8mm
- Refracting power (1/f) 60-68 diopters (1 diopter = 1m\(^{-1}\))
- Macula lutea: region at center of retina
- Blind spot: where ganglion cell axons exit retina from the optic nerve

http://www.cas.vanderbilt.edu/bsci111b/eye/human-eye.jpg