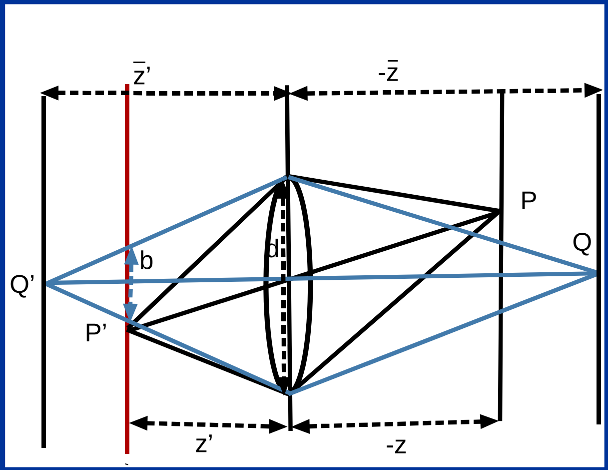
Image formation 2

Blur circle

Points a t distance $-z$ are brought into focus at distance z'

A point at distance $-\bar{z}$ is imaged at point $\bar{z}'$ from the lens

$$\frac{1}{\bar{z}'} + \frac{1}{-\bar{z}} = \frac{1}{f}$$



and so

$$\bar{z}' - z' = \frac{f}{(\bar{z}+f)}\frac{f}{(z+f)}(\bar{z}-z)$$

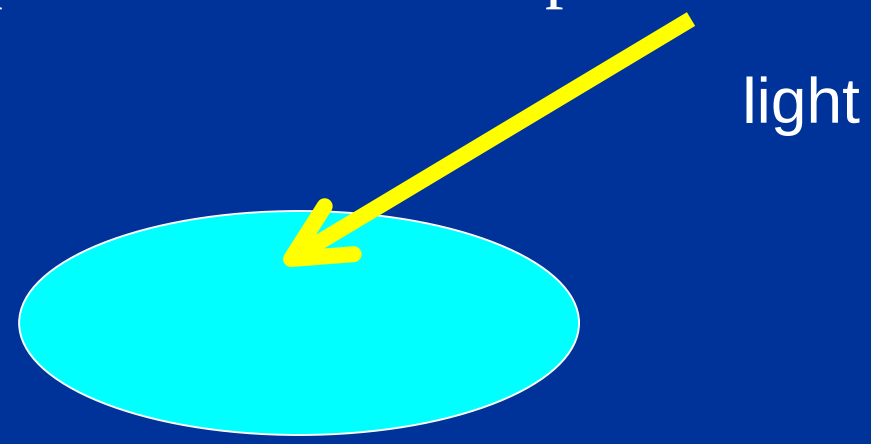
Thus points at distance $-\bar{z}$ will give rise to a blur circle of diameter

$$b = \frac{d}{\bar{z}'}\,|\,\bar{z}' - z'|$$

with d the diameter of the lens

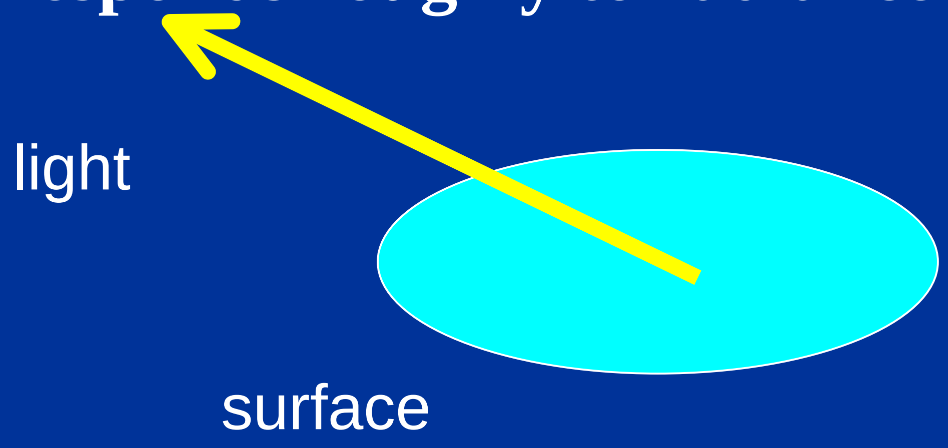
Irradiance, E

- Light power per unit area (watts per square meter) incident on a surface.
- If surface tilts away from light, same amount of light strikes bigger surface (less irradiance)(no foreshortening)
- E times pixel area times exposure time -> pixel intensity



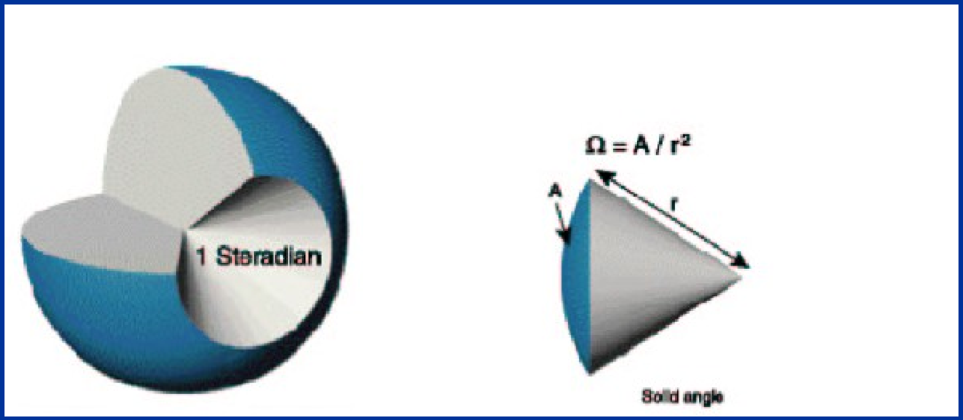
Radiance, L

- Amount of light radiated from a surface into a given solid angle per unit area (watts per square meter per steradian).
- Note: the area is the foreshortened area, as seen from the direction that the light is being emitted.
- **Brightness corresponds roughly to radiance**

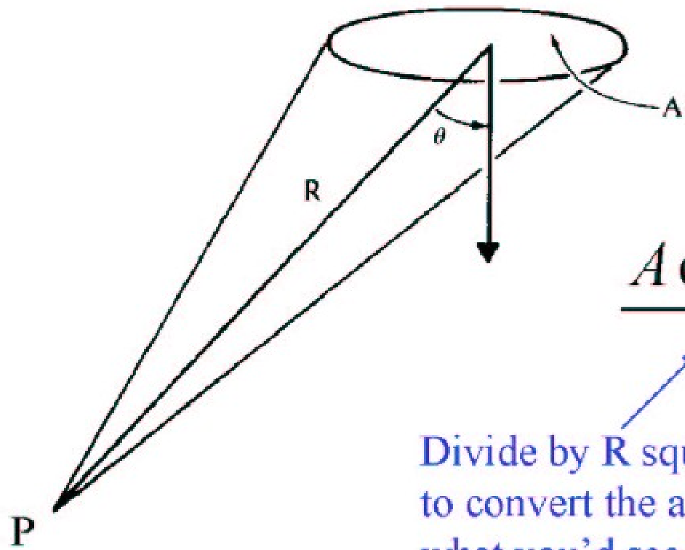


Solid angle

- The solid angle subtended by a cone of rays is the area of a unit sphere (centered at the cone origin) intersected by the cone
- A hemisphere cover 2π sterradians



What's the solid angle subtended by this patch, area A, seen from P?

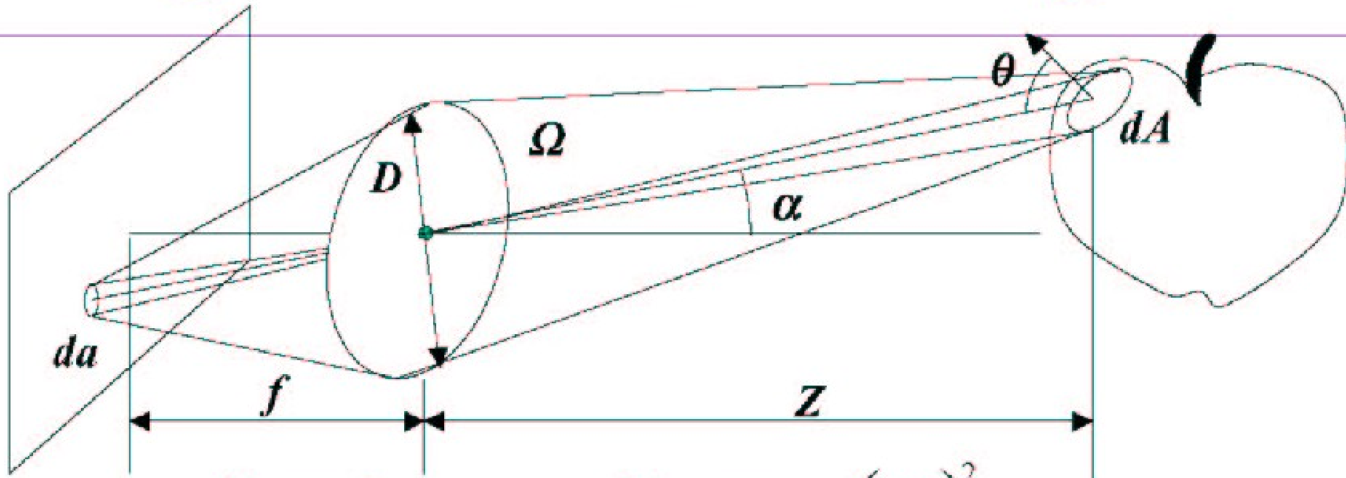


Multiply by cos(θ) to account for foreshortening

$$\frac{A\cos(\theta)}{R^2}$$

Divide by R squared to convert the area to what you'd see on a unit sphere

Pixel Brightness and Scene Brightness



$$d\omega = \frac{da\cos\alpha}{(f/\cos\alpha)^2} = \frac{dA\cos\theta}{(Z/\cos\alpha)^2} \Longrightarrow \frac{dA}{da} = \frac{\cos\alpha}{\cos\theta}\left(\frac{Z}{f}\right)^2$$

$$\Omega = \frac{\pi}{4}\frac{D^2\cos\alpha}{(Z/\cos\alpha)^2} = \frac{\pi}{4}\left(\frac{D}{Z}\right)^2\cos^3\alpha$$

$$dP = L\,dA\,\Omega\cos\theta \Longrightarrow dP = L\,dA\frac{\pi}{4}\left(\frac{D}{Z}\right)^2\cos^3\alpha\cos\theta$$

Power emitted from patch DA

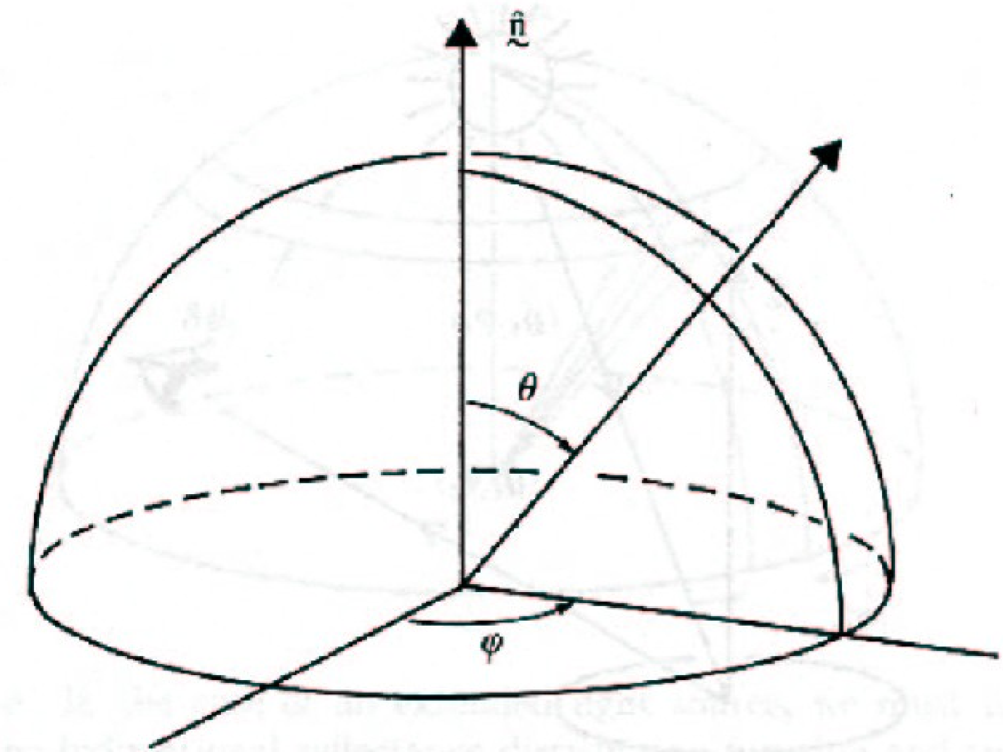
$$E = \frac{dP}{da} = L\frac{dA}{da}\frac{\pi}{4}\left(\frac{D}{Z}\right)^2\cos^3\alpha\,\cos\theta \Longrightarrow E = \frac{\pi}{4}\left(\frac{D}{f}\right)^2\cos^4\alpha\;L$$

$$E = k\,L$$

Relationship :Image Irradiance and Scene Radiance

$$E = L\frac{\pi}{4}\left(\frac{D}{f}\right)^2 \cos^4\alpha$$

Coordinate system



Horn, 1986

Radiosity

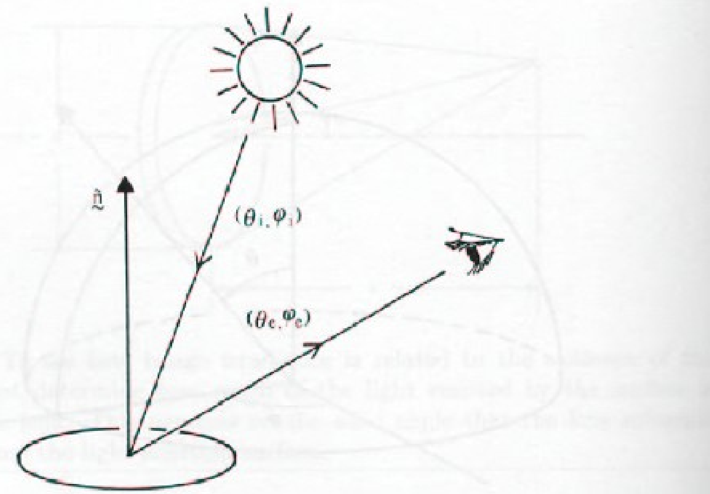
The total power leaving a point on a surface per unit area on the surface

$$B(P) = \int_{\Omega} L(P, \theta, \phi) \cos\theta d\Omega$$

If radiance independent of angle -> ingegrate over hemisphere

$$B(P) = L(P) \int_{0}^{\frac{\pi}{2}} \int_{0}^{2\pi} \cos\theta \sin\theta d\phi d\theta = \pi L(P)$$

BRDF



Horn, 1986

Figure 10-7. The bidirectional reflectance distribution function is the ratio of the radiance of the surface patch as viewed from the direction (θ_e, ϕ_e) to the irradiance resulting from illumination from the direction (θ_i, ϕ_i).

$$BRDF = f(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{L(\theta_e, \phi_e)}{E(\theta_i, \phi_i)}$$

unit: sr^{-1}

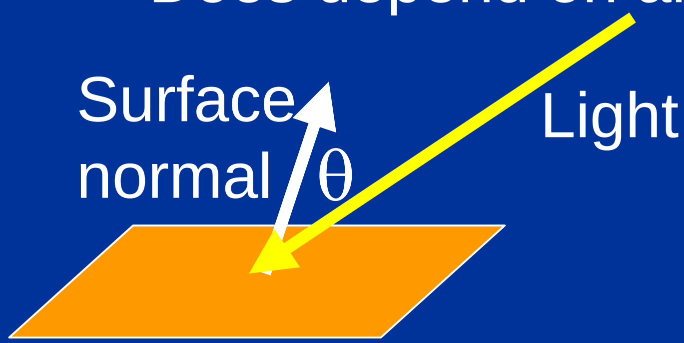
$$= \frac{L_e(\theta_e, \phi_e)}{L_i(\theta_i, \phi_i) \cos\theta_i d\omega}$$

Special Cases: Lambertian

$$f(\theta_i, \phi_i, \theta_e, \phi_e) = \rho \frac{1}{\pi}$$

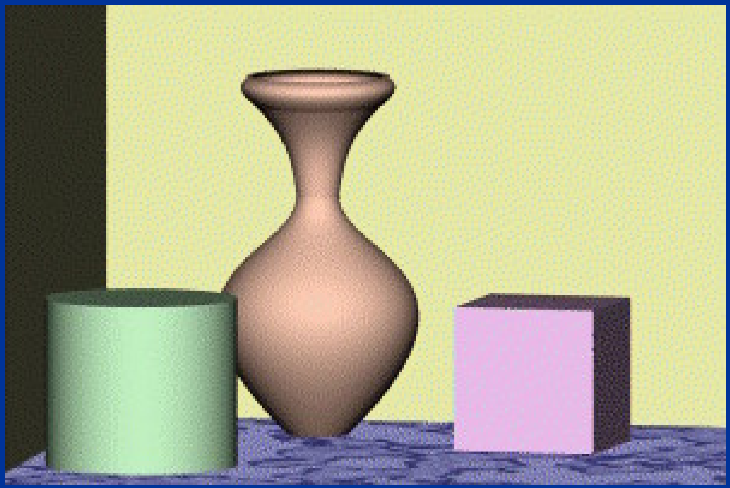
Note: reflected light is with strength proportional to cos of angle with surface normal, but the area is foreshortened

- Albedo is fraction of light reflected.
- Diffuse objects (cloth, matte paint).
- Brightness doesn't depend on viewpoint.
- Does depend on angle between light and surface.



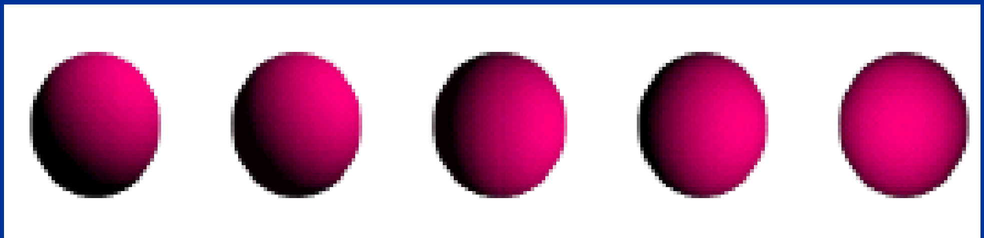
$$L(\theta_e, \phi_e) \propto \cos(\theta)$$

Lambertian Examples



Scene

(Oren and Nayar)

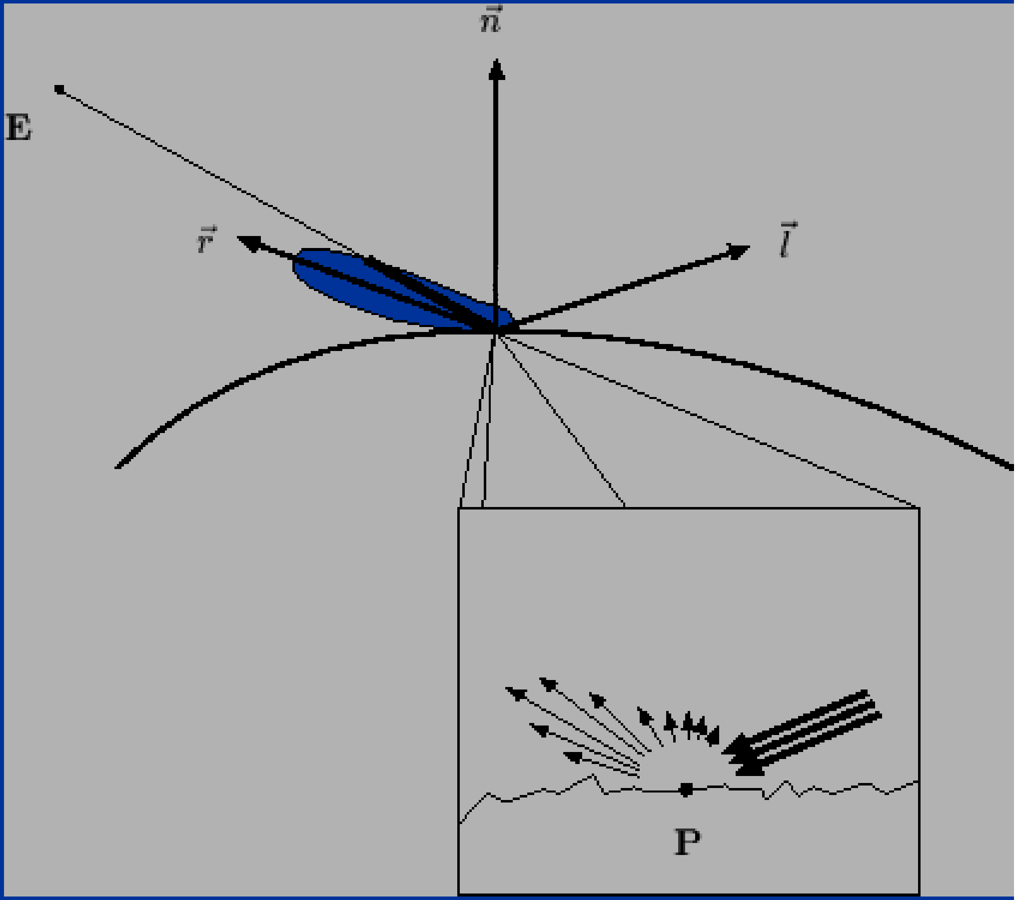


Lambertian sphere as the light moves.

(Steve Seitz)

Specular surfaces

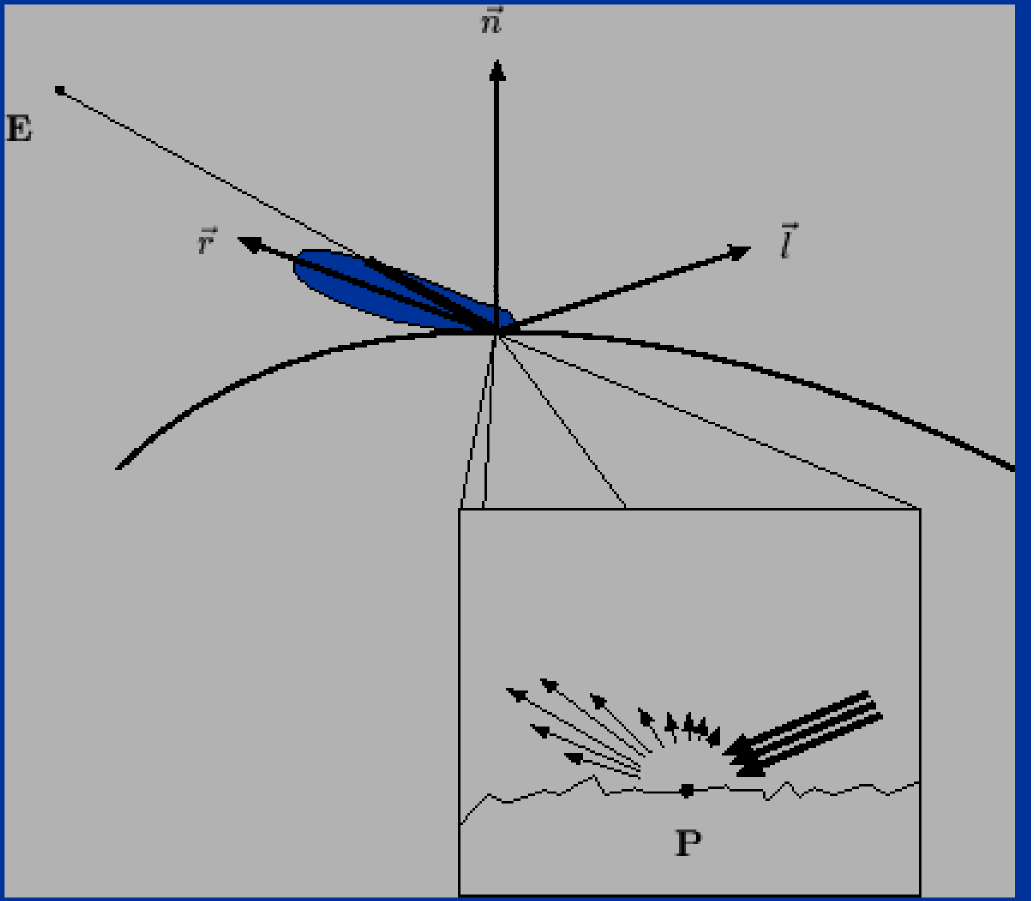
- Another important class of surfaces is specular, or mirror-like.
  - radiation arriving along a direction leaves along the specular direction
  - reflect about normal
  - some fraction is absorbed, some reflected
  - on real surfaces, energy usually goes into a lobe of directions



(http://graphics.cs.ucdavis.edu/GraphicsNotes/Shading/Shading.html)

Specular surfaces

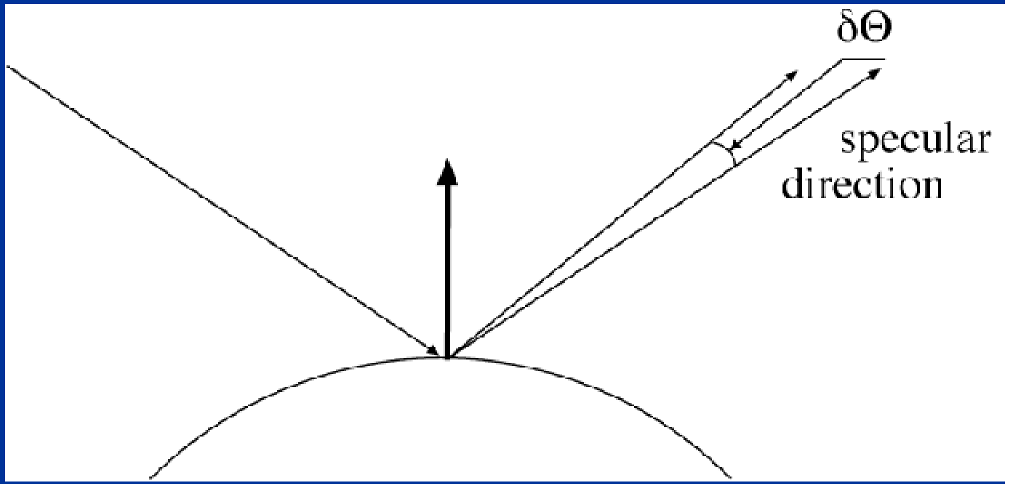
•Brightness depends on viewing direction.



(http://graphics.cs.ucdavis.edu/GraphicsNotes/Shading/Shading.html)

Phong's model

- Vision algorithms rarely depend on the exact shape of the specular lobe.
- Typically:
  - very, very small --- mirror
  - small -- blurry mirror
  - bigger -- see only light sources as "specularities"
  - very big -- faint specularities



(Forsyth & Ponce)

- Phong's model
  - reflected energy falls off with $\cos^n(\delta\vartheta)$

Lambertian + Specular Model

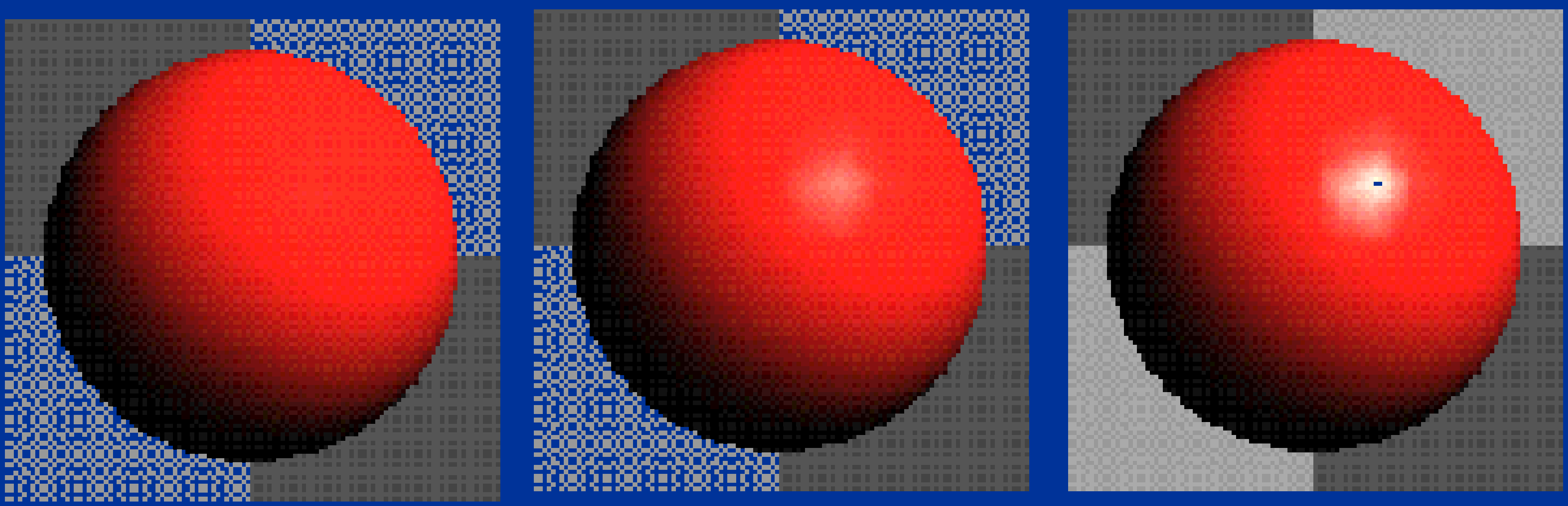
$$L(P,\theta_o,\phi_o) = \rho_d(P)\int_{\Omega} L(P,\theta_i,\phi_i)\cos\theta_i d\Omega$$

$$+\ \rho_s(P)L(P,\theta_s,\phi_s)\cos^n(\theta_s - \theta_o)$$

Lambertian + specular

- Two parameters: how shiny, what kind of shiny.
- Advantages
  - easy to manipulate
  - very often quite close true
- Disadvantages
  - some surfaces are not
    - e.g. underside of CD's, feathers of many birds, blue spots on many marine crustaceans and fish, most rough surfaces, oil films (skin!), wet surfaces
  - Generally, very little advantage in modelling behaviour of light at a surface in more detail -- it is quite difficult to understand behaviour of L+S surfaces (but in graphics???)

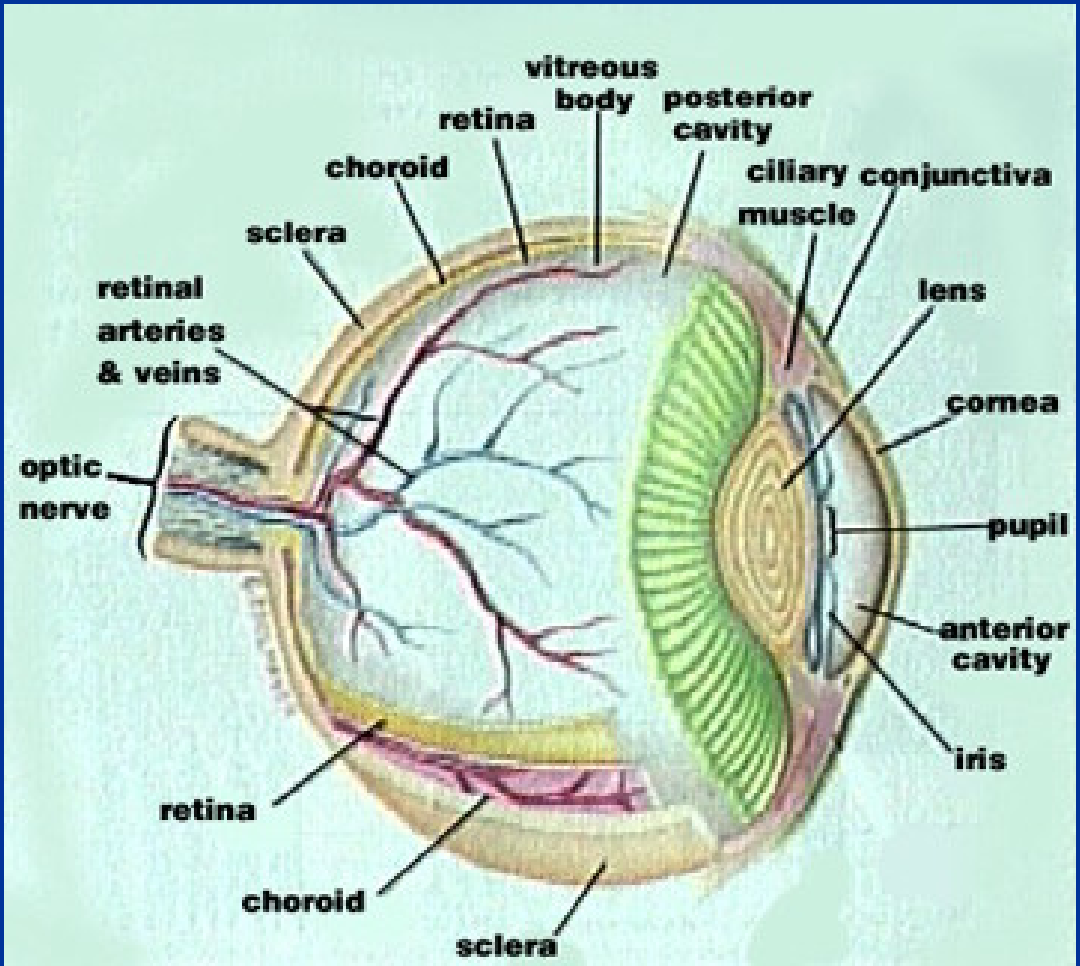
Lambertian+Specular+Ambient



(http://graphics.cs.ucdavis.edu/GraphicsNotes/Shading/Shading.html)

Human Eye

- pupil: 1-8mm
- Refracting power (1/f) 60-68 diopters (1 diopter = 1m$^{-1)}$
- Macula lutea: region at center of retina
- Blind spot: where ganglion cell axons exit retina from the optiv nerve



http://www.cas.vanderbilt.edu/bsci111b/eye/human-eye.jpg