Problem 1. Discrete time Fourier transform (DTFT) properties:

(a) What does it mean for the DTFT to converge?

(b) If a signal is real-valued, what does this tell you about the DTFT?

(c) Why is the DTFT a periodic function of frequency?

Problem 2. Determine the DTFT of the sequence

\[ y[n] = (n + 1)\alpha^n u[n], \quad |\alpha| < 1. \]

Problem 3. A discrete-time causal, linear, time-invariant system is described by the following input-output equation:

\[ 6y[n] - 5y[n - 1] + y[n - 2] = 6x[n]. \]

Here, \( x \) is the input to the system and \( y \) is the corresponding output.

(a) Find the frequency response of this system.

(b) Find the impulse response of this system.

(c) Find the response of this system to the input signal \( x_1[n] = \delta[n] - \frac{1}{3}\delta[n - 1] \).

(d) Find the response of this system to the input signal \( x_2[n] = e^{j\pi n} \).

Problem 4. Let

\[ x[n] = n2^{-|n-3|} \]

for all integer \( n \). Find the DTFT of \( x \).

Problem 5. The exponential decaying sequence

\[ x[n] = a^n u[n], \quad 0 \leq a \leq 1, \]

is applied at the input of an LTI system with the impulse response

\[ h[n] = b^n u[n], \quad 0 \leq b \leq 1. \]

Using the DTFT approach, calculate the output of the system.
Problem 6. DTFT for DT periodic signals:

The DTFT $X(e^{j\omega})$ of a periodic sequence $x[n]$ with period $N$ is given by

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} 2\pi a_k \delta \left( \omega - \frac{2\pi k}{N} \right),$$

where $a_k$ are the DTFS coefficients of $x[n]$. The DTFS coefficients are given by

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk2\pi n}.$$

Calculate the DTFT of

$$x[n] = 3\sin \left( \frac{2\pi}{7} n + \frac{\pi}{4} \right).$$

Problem 7. Oppenheim & Willsky, problem 5.23.

Problem 8. Oppenheim & Willsky, problem 5.33 a, b, c-i, c-ii.

Problem 9. Oppenheim & Willsky, problem 5.34.

